

Grades K-5

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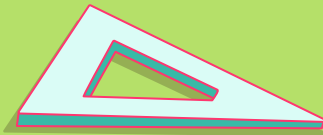
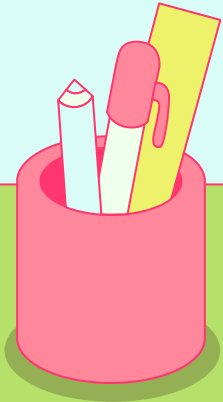
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# Elementary Math FRAMEWORK

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Presented By:  
Mrs. Erika Kelly; PreK-6 Applied Sciences  
Supervisor





# TABLE OF CONTENTS



01

## Structure

Math Workshop Model &  
CRA Approach



02

## Resources

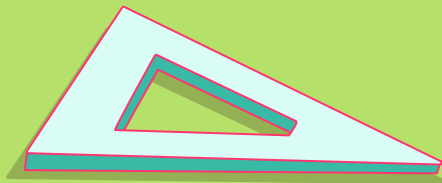
Supplemental Math Series:  
Reveal Math & Number  
Worlds



03

## 3 Year Plan

Focus on Continued  
Growth



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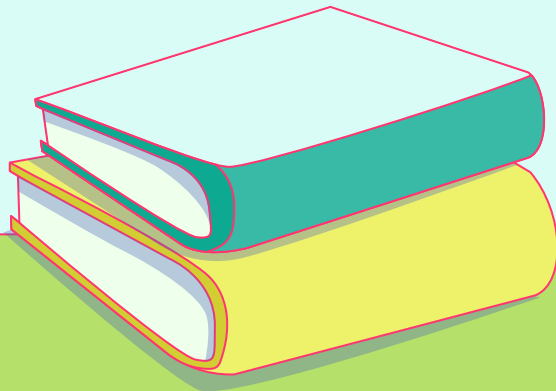
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# STRUCTURE

The Math Workshop Model & Infusion of the  
CRA Approach



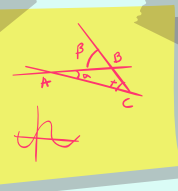
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$$\sin(-\alpha) = -\sin \alpha$$

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos \alpha$$

$$a^2 + b^2 = c^2$$

# Why Math Workshop?



## Differentiation

Guides teachers in differentiating instruction in meaningful ways

## Activities

Students experience various types and forms of activities relating to the overarching learning objective



## Structure

Students are able to learn from their teachers and peers in small flexible groups

## Assessment

Students authentically partake in the assessment process through self-assessing and tracking their own progress



**It fits with any supplemental Math series!**

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# Math Workshop Model

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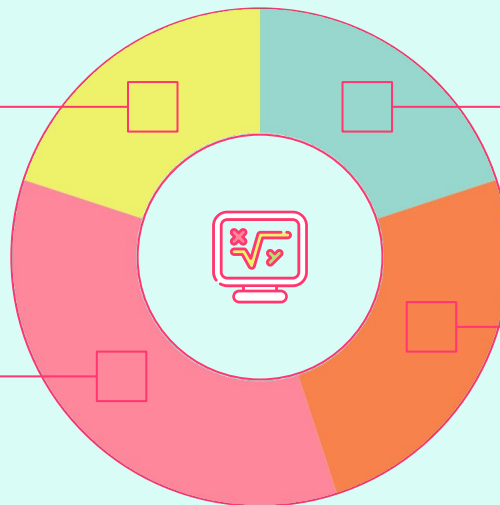
## Warmup

Number Sense & Fact  
Fluency Routines

25%

## Mini Lesson

Direct Instruction: Skill  
Based/Conceptual



55%

## Centers

Differentiated Work  
Centers & Small Group  
Instruction

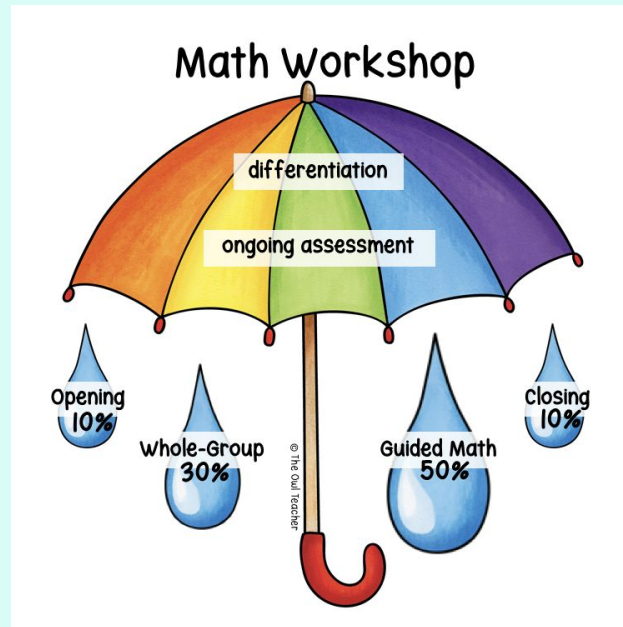
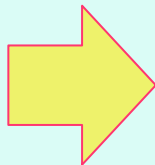
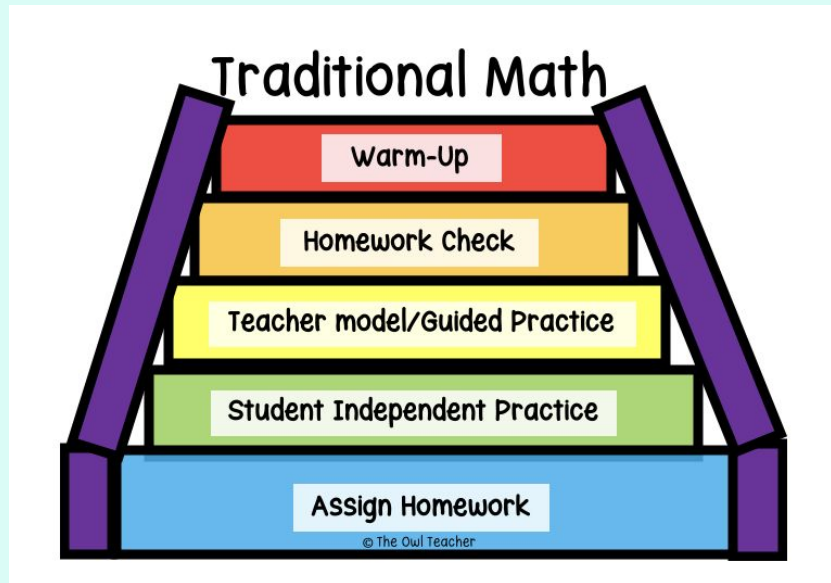
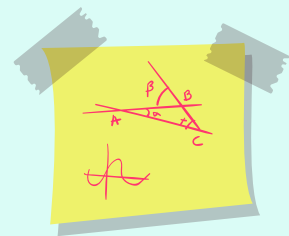
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## Closure

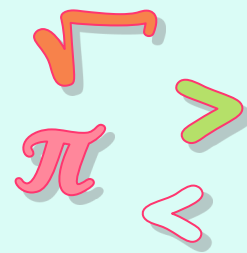
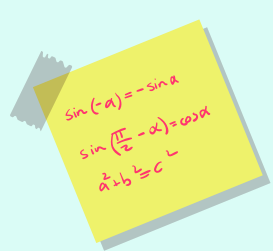
Assessment & Share



# Math Workshop Model



All Percentages can differentiate



# Warmup

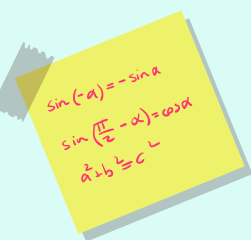
**“Fact fluency is not an add-on—it’s an integral part of learning arithmetic with deep understanding.”**

- Graham Fletcher & Tracy Johnston Zager

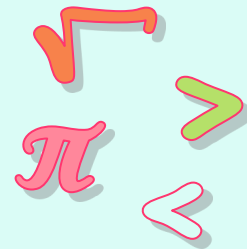
The Warmup component of the math workshop model solely focuses on the realm of Number Sense and Fact Fluency. The following activities are designed to be utilized during the warmup component to address the vast intricacies of developing students Number Sense and Fact Fluency:

- Graham Fletcher Kit:
  - Image Talk
  - Tool Talk
- Be Curious
- Notice and Wonder
- Number Sense Routine:
  - Find the Pattern, Make the Pattern
  - Would You Rather?
  - Let's Count
  - Mystery Number
- Esti-mystery




$$\sin(-a) = -\sin a$$
$$\sin\left(\frac{\pi}{2} - a\right) = \cos a$$
$$a^2 + b^2 = c^2$$

# Mini Lesson



**Length of the mini lesson:** 10-15 minutes

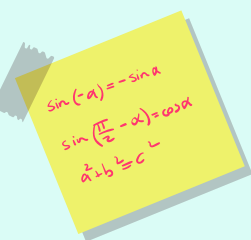
The mini lesson component of the math workshop model is targeted and standards/skills driven, while also goal-oriented around the objective of the lesson and what students will learn. The mini lesson is constructed based on the starting point of the grade level Mathematics NJSL standards, then developed with application of the Concrete-Representational-Abstract (CRA) approach and various resources/tools from supplemental programs and digital platforms.

**Essential elements of the mini lesson:**

- Students engage in discussions to talk about their mathematical thinking and reasoning
- Teachers act as facilitators
- The teacher models explicitly
- Incorporates the Concrete-Representational-Abstract (CRA) approach
- Students have the opportunities to practice the concept/skill using the appropriate and applicable mathematical tools
- Embeds and concludes with formative assessment approaches

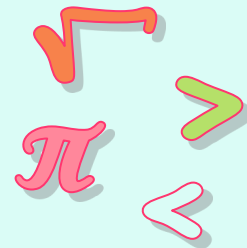






$\sin(-a) = -\sin a$   
 $\sin\left(\frac{\pi}{2} - a\right) = \cos a$   
 $a^2 + b^2 = c^2$

# Centers & Small Group Instruction



Following their daily math lesson, students have an extended work session when they practice the new math skills taught that day, build their problem solving skills by solving performance tasks, get hands-on math practice, work on project based learning, complete assessments, use technology to practice math skills, and receive differentiated instruction from you.

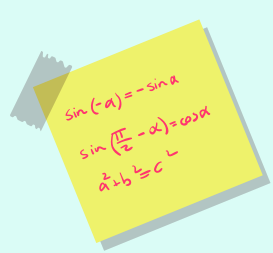
## Centers

- Students rotate through various center activities relating to the lessons objective
- Center work is differentiated and tiered to meet the needs of all individuals
- Each center activity incorporates the various levels of depth of knowledge (DOK)
- **Traditional Approach:**
  - **M:** Meet the Teacher (small group)
  - **A:** At your Desk (independent)
  - **T:** Technology
  - **H:** Hands on (manipulatives/games)

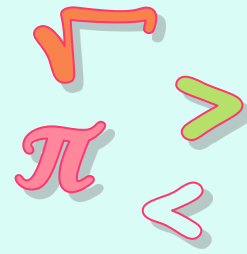
## Small Group Instruction

- Comprised of flexible groups: based on data from assessments
- **Two approaches:** skill based (standard) versus strategy based (fluency)
- All instruction relates to the lessons objective and extends or enriches the lesson (if skill based)
- Embeds the elements of the CRA model
- Could also be utilized to support PBL and conferences
- Embeds formative and summative assessment approaches





# Closure



Students take a moment to reflect on the math progress made that day and share their thoughts verbally, or in writing. This is the second brief time when all students are potentially working on the same activity simultaneously, while readdressing the lessons objective. A few methods for sharing include:

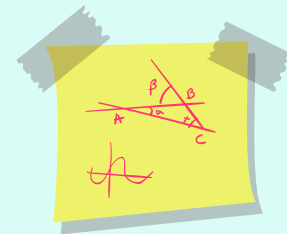
- Think, Pair, Share
- Exit Ticket
- Whole class share out
- Self-reflection activity



A pink pi symbol ( $\pi$ ) with a shadow.

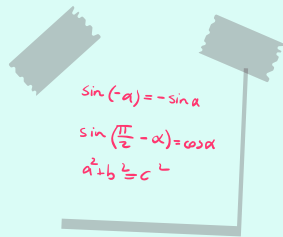
An orange plus sign (+) with a shadow.

A yellow number 2 with a shadow.



A pink number 0 with a shadow.

An orange number 5 with a shadow.



“All students have ideas about math that are valid and worth talking about.”

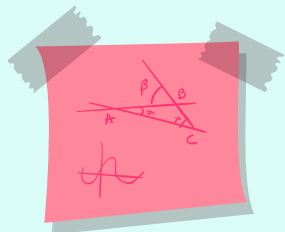
A pink sigma symbol ( $\Sigma$ ) with a shadow.

**—Annie Fetter**

A green percent sign (%) with a shadow.

A yellow greater-than-or-equal-to symbol ( $\geq$ ) with a shadow.

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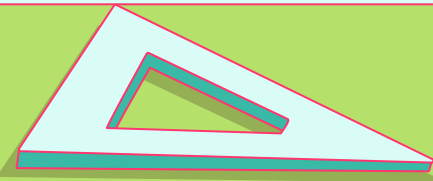
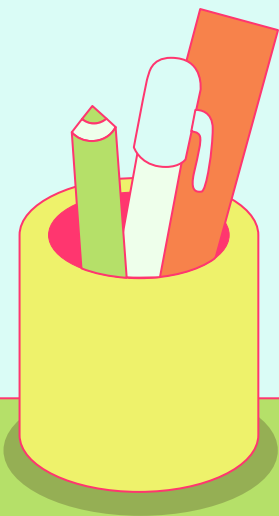
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# Resources

Supplemental Math Curriculum Series: Reveal  
Math & Number Worlds





$\sin(-a) = -\sin a$   
 $\sin\left(\frac{\pi}{2} - a\right) = \cos a$   
 $a^2 + b^2 = c^2$

# Welcome to Reveal Math K-5



Reveal Math is one of the curriculum series utilized across grades K-5 in Mathematics. This program is a supplemental resource to the Elementary Mathematics curriculum. The instruction that students receive on a daily basis are built on the foundation of the NJSL standards, and supplemented with resources from the program, digital platforms, etc.

**McGraw Hill has worked to construct Reveal Math to incorporate the following goals:**

- The lesson model offers two instructional options for each lesson; one being a guided exploration that is teacher-led and the other being an activity-based exploration that has students exploring concepts through small group activities.
- The lesson model incorporates an initial sense-making activity that builds students' proficiency with problem solving. By constructing lessons to focus on sense-making, students are able to develop and refine their questioning and reasoning skills.
- The lessons focus on fostering mathematical language and rich mathematical discourse.
- The Math Is... unit builds student agency for mathematics. Students consider their strengths, thinking habits, and classroom norms to be successful within math.






$\sin(-a) = -\sin a$   
 $\sin\left(\frac{\pi}{2} - a\right) = \cos a$   
 $a^2 + b^2 = c^2$

# Reveal Math Authorships



The following expert authors participated in the formulation of the program:

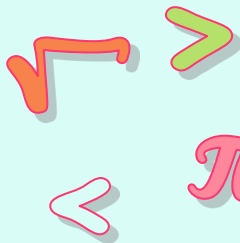
*\*Any authors that are italicized are ones that have been used by the teachers of Roxbury Public Schools for several years*

- **Ralph Connelly, Ph.D:** Development of early mathematical understanding
  - **Annie Fetter:** Fostering strong problem solvers
  - **Linda Gojak, M.Ed:** Theory and practice of strong mathematics instruction
  - **Sharon Griffin, Ph.D:** Number sense and the achievement of students
  - **Susie Katt, M.Ed:** Advocate for unique needs of our youngest mathematicians
  - **Ruth Harbin Miles, Ed.S:** Developing teachers' math content and strategy knowledge
  - *Nicki Newton, Ed.D: Expert in bringing student-focused strategies and workshops into the classroom*
  - *John SanGiovanni, M.Ed: Leader in understanding the mathematics needs of students and teachers*
  - **Raj Shah, Ph.D:** Perseverant problem-solvers and student curiosity in mathematics
  - **Jeff Shih, Ph.D:** Advocate for the importance of student knowledge
  - *Cheryl Tobey, M.Ed: Facilitator of strategies that drive informed instruction decisions*
  - **Dinah Zike, M.Ed:** Creator of learning tools that make connections through visual-kinesthetic techniques
- 

$$\sin(-\alpha) = -\sin \alpha$$

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos \alpha$$

$$a^2 + b^2 = c^2$$



# Reveal Math Is....

Focused on the continued growth of the whole child in Mathematics:

**Math Is.... Unit:** *Focuses on the mathematical practices and builds students as mathematicians*

- Promotes students finding success in math and becoming doers of math
- Promotes students applying learned practices to problem solve

**Mathematical Objectives:**

- Content Objective
- Language Objective
- Social Emotional Objective

**Social Emotional Learning (SEL) Integration:**

- Based on CASEL Social Emotional Learning competencies
- Math Is... mindset prompts

## Unit 1 Math Attitude Survey

Name \_\_\_\_\_

- I can see math in the world around me.  
I agree ☐ I'm not sure ☐ I disagree ☐
- Math is about explaining your thinking.  
I agree ☐ I'm not sure ☐ I disagree ☐
- Math is about solving equations quickly.  
I agree ☐ I'm not sure ☐ I disagree ☐
- I am good at math.  
I agree ☐ I'm not sure ☐ I disagree ☐
- Math is something I will need when I grow up.  
I agree ☐ I'm not sure ☐ I disagree ☐
- Everyone can be good at math.  
I agree ☐ I'm not sure ☐ I disagree ☐

### Focus

#### Content Objectives

- Students represent multiplication as comparison.
- Students represent multiplicative comparison statements as multiplication equations.

#### Language Objectives

- Students compare quantities using multiplication and use multiplication equations to represent multiplicative comparison statements using the expression *times as much as*.
- To optimize output, ELs participate in MLR7: Compare and Connect.

#### SEL Objective

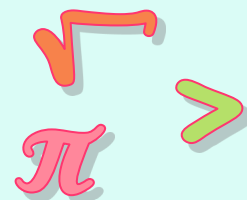
- Students demonstrate self-awareness of personal strengths and areas of challenge in mathematics.

### SEL Self-Awareness: Accurate Self-Perception

As students begin to think about multiplicative comparison in the Notice & Wonder routine, encourage them to make connections to concepts they are more familiar or comfortable with, such as comparing numbers. They can also use more familiar strategies to check their answers. As students continue to compare numbers using multiplication, differentiate instruction to provide opportunities for students to experience success and gratification as well encounter appropriate amounts of productive struggle.



# Reveal Math: Academic Focus



## The supplemental curriculum series includes the following:

- **Learning Targets**
- **Standards Alignment:** NJSLs
- **Vocabulary:** Based on researched tiers of vocabulary instruction
  - Mathematical terms
  - Academic terms
- **Coherence:** RTI approach
  - Previously learned (relates to prior grade level standard/prerequisite skill)
  - Current (relates to grade level standard)
  - Future (relates to next grade level standard)
- **Rigor:** Informs teachers of the rigor of the lesson
  - Conceptual Understanding
  - Procedural Fluency
  - Real World Application

### Coherence

#### What Students Have Learned

- Students made drawings and wrote equations to solve word problems involving multiplication and division facts within 100. (Grade 3)
- Students determined the unknown whole number in a multiplication or division equation relating three whole numbers. (Grade 3)

#### What Students Are Learning

- Students interpret a multiplication equation as a comparison and represent verbal statements of comparison as multiplication equations.
- Students distinguish between additive and multiplicative comparisons.
- Students write multiplication and division equations with an unknown to represent and solve multiplicative comparison problems.

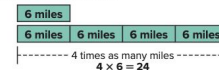
#### What Students Will Learn

- Students interpret multiplication as scaling by explaining the size of a product in comparison to the size of the factors. (Grade 5)
- Students fluently multiply multi-digit whole numbers using the standard algorithm. (Grade 5)
- Students interpret a fraction as division of the numerator by the denominator. (Grade 5)

### Focus

#### Multiplicative Comparison

Students use multiplication and division to translate phrases such as *times as many* and *times as much* to find the total or unknown factors in problems. They recognize that these kinds of phrases are associated with *multiplicative comparisons*. One commonly used representation for such comparisons is a bar diagram.



Students will further understand that this and other representations of multiplicative comparison can also be used for division.

They understand this because of the relationship between multiplication and division. In solving multiplicative comparison problems, students use this relationship to write and solve multiplication and division equations with the unknown in any position. Strictly speaking, the phrases “times as great as” and “times greater than” do not mean the same thing. For example, the number 30 is 5 times as great as 6. However, 36 is 5 times greater than 6. This is because “5 times greater than 6” means the answer must be  $5 \times 6$ , or 30, greater than 6. The equation,  $6 + 30 = 36$ , shows that 36 is 30 greater than 6. Despite the formal distinction, popular media frequently use “times greater than” to mean the same thing as “times as great as.” Make sure students are aware of and capable of working with this usage.

#### Language of Math

##### Vocabulary

Students will be using these key terms in this unit:

- **Additive comparison** (Lesson 4-2): It is the action or result of finding how much more or less one number is than another. These problems can be solved by using addition or subtraction. Students should be familiar with solving additive comparison problems from Grade 2, but may not have been introduced to the term.
- **Multiplicative comparison** (Lesson 4-3): It is the action or result of finding how many times as much one number is compared to another. These problems can be solved by using multiplication or division. Students should be given opportunities to write their own multiplicative comparison problems.

• **Unknown** (Lesson 4-3): Students were introduced to this term in the context of equations in Grade 1. It is a symbol (such as a letter, box, or ?) that represents a value that needs to be determined. When the correct number replaces an unknown in an equation, the equation is true. “This is a new term.”

#### Math Language Development

##### A Focus on Speaking

A deliberate effort should be made to engage students in speaking mathematically. Discourse is cited as a critical component in our effort to give students more ownership of their learning.

Teachers are called to probe student thinking and elicit verbal responses, using open-ended questions. This allows students to dig deeper into their thoughts and their abilities to verbalize them. By speaking their thoughts, students process their understanding more thoroughly. This helps them understand more clearly, and builds their recall of concepts.

In this unit, pay attention to opportunities to elicit verbal responses from students.

- Ask students to explain their representation of a problem as they share it with you. Math is different from everyday language in that with math, we are accustomed to show our thinking. We are inclined, for example, to write equations or draw pictures or graphs to explain our ideas.
- Have students identify equations they can use in different situations and explain the equations in multiple ways using different wording—for example, multiplied by, times as many as, groups of, equal to, times less than, and so on. Sometimes verbalizing an equation brings out, or speaks, its meaning more effectively than simply writing it.
- For students who have difficulty explaining a concept, problem, or representation, you may tell them some or all the mathematical and academic vocabulary that you would expect them to use in their explanations.

### Rigor

#### Conceptual Understanding

Students develop understanding of:

- using multiplication to compare quantities;
- representing comparisons using physical manipulatives, diagrams, and equations;
- comparison problems as they distinguish between additive and multiplicative comparison.

#### Procedural Skill and Fluency

Students build proficiency with:

- using addition and multiplication skills to solve additive and multiplicative comparison problems;
- using multiplication and division skills to solve multiplicative comparison problems.

*Procedural skill and fluency is not a targeted element of rigor for the standards in this unit.*

#### Application

Students apply their knowledge of:

- additive and multiplicative comparison to solve real-world problems;
- multiplication and division to solve real-world problems involving multiplicative comparison.



# Reveal Math: Academic Focus

The supplemental curriculum series includes the following:

- **Lesson Structure**
  - **Guided Exploration:** Teacher directed
  - **Activity Based Exploration:** Student discovery based
- **Practice**
  - **On my Own:** Tiered independent practice
  - **Math Replay:** Videos to support students in revisiting the concept and methodology of instruction based on the concept
  - **Additional Practice:** Tiered additional practice (i.e. for homework)

## Number Routines

**Build Fluency** The number routines found at the beginning of each lesson help students build number sense and operational fluency. They also help students develop the thinking habits of mind that are important for proficient doors of math.

### Find the Missing Values

**Purpose:** Build identification of patterns and efficiency with solving equations while examining a list of related equations.

**Overview:** Students analyze a series of equations to look for patterns that they can use to determine the missing values in the equations. As students share their solutions, the teacher can reveal the missing values.

### Can You Make the Number?

**Purpose:** Build flexible thinking and efficiency with operations.

**Overview:** Students use all the given numbers to build expressions with a value matching the target number. The teacher records students' expressions, then facilitates a discussion about students' expressions.

### Greater Than, Less Than

**Purpose:** Build proficiency with number and place value sense; estimating and comparing skills.

**Overview:** Students use mental math to estimate or evaluate the value of given expressions and then compare the value of the expressions to a target benchmark number. Students share their solutions and thinking.

## Additional Practice

Name \_\_\_\_\_

### Review

You can compare two quantities by telling how many more or how many times as many.

Nick hikes 2 miles. Logan hikes 4 more miles than Nick. Ava hikes 3 times as many miles as Nick. How many miles do Logan and Ava hike?

2 miles	2 miles	2 miles	2 miles
Logan's Hike	Ava's Hike	Nick's Hike	

2 + 4 = Logan's miles  
2 + 4 = 6  
Logan hikes 6 miles.

2 x 3 = Ava's miles  
2 x 3 = 6  
Ava hikes 6 miles.

Choose the correct comparison statement.

1. A small froggie is 15 feet tall. A taller froggie is 4 times as tall as the small froggie. How tall is the taller froggie?

15 + 4 = ?  
15 x 4 = ?

2. Avery's green lizard eats 5 crickets. Her brown lizard eats 6 more crickets than her green lizard. How many crickets does her brown lizard eat?

5 + 6 = ?  
5 x 6 = ?

## On My Own

Name \_\_\_\_\_

What equation can you write to represent and solve the comparison?

- 8 more than 4  
**Sample answer:**  $4 + 8 = 12$
- 3 times as many as 5  
**Sample answer:**  $3 \times 5 = 15$
- 2 times as long as 9 feet  
**Sample answer:**  $2 \times 9 = 18$
- 5 times as far as 10 miles  
**Sample answer:**  $5 \times 10 = 50$

How can you represent the problem? Draw a bar diagram and write an equation to solve.

- A small bridge is 40 feet long. A new bridge is 3 times as long as the small bridge. How long is the new bridge?  
**120 feet; Sample answer:**  $3 \times 40 = 120$ ;  
**Check students' drawing.**
- Raya has 8 pencils in her school box. Miranda has 4 more pencils than Raya. How many pencils does Miranda have?  
**12 pencils; Sample answer:**  $8 + 4 = 12$ ;  
**Check students' drawing.**
- Loukia is 5 feet tall. The tree in her backyard is 4 times as tall as Loukia. How tall is the tree?

**20 feet; Sample answer:**  $4 \times 5 = 20$ ;  
**Check students' drawing.**

Unit 4 • Multiplication in Comparison 111

## Sense-Making Routines

**Notice & Wonder:** What do you notice? What do you wonder? (Lesson 7-1) Students are presented with 5 bills and 2 piggy banks with the mathematical goal of focusing students on equal sharing. Students share things they noticed and reflect on what they wonder about the situation.

**Notice & Wonder:** How are they the same? How are they different? (Lesson 7-2) Students use three equations with dividends that are multiples of 25. This gets students thinking about the relationship between dividends and divisors in a set of equations. This helps students prepare to learn about using compatible numbers to estimate quotients.

**Notice & Wonder:** Tell me everything you can. (Lesson 7-3) Students are presented with an image of 85 stickers. The purpose of this routine is to get students thinking about equal sharing.

**Notice & Wonder:** Tell me everything you can. (Lesson 7-4) Students are presented with a rectangle labeled as 246 square feet and divided into 3 equal sections. This routine gets students thinking about ways to divide a three-digit dividend by a 1-digit divisor.

**Notice & Wonder:** How are they the same? How are they different? (Lesson 7-5) This routine uses two area models that show different ways to divide the same area. Students think about different ways to use partial quotients to find a quotient.

**Notice & Wonder:** What could the question be? (Lesson 7-6) Students are presented with an image of 4 people and 5 carrots with the goal of getting students to think about how the carrots will be shared.

**Notice & Wonder:** What could the question be? (Lesson 7-7) Students are presented with an image of 4 robots each requiring 4 batteries and some batteries with the mathematical goal of getting students to interpret remainders in the context of a problem.

**Numberless Word Problem:** What math do you use? (Lesson 7-8) Students are presented with a word problem in which some flowers are given away and the rest are divided. This encourages students to think about the steps in a multi-step equation.

## CHOOSE YOUR OPTION

### Activity-Based Exploration

Students develop understanding of multiplicative comparison by creating their own comparison statements. They use models and equations as representations of the multiplicative relationship.

**Materials:** counters, connecting cubes, Number Cards 0-10 Teaching Resource

**Directions:** Students work in pairs, taking turns in each role. One partner chooses a number card and uses this number to determine the number of connecting cubes to create a stick or the number of counters in a set. The second partner chooses another number card and uses this number to determine the number of sticks of connecting cubes or a number of groups of counters. For example, a student makes a stick of 3 connecting cubes. The other student makes 4 sticks of 3 connecting cubes. Then students discuss the comparison between the number of connecting cubes in one stick and the number of cubes in all 4 sticks, and work together to write a comparison statement and an equation to show the relationship.

### Support Productive Struggle

- What are some ways you can represent the comparison?
- How can you state the comparison between numbers of cubes in one stick and numbers of cubes in all \_\_\_\_\_ sticks?
- What types of equations can be used to show the comparison?

### Math Is... Thinking

How does the equation show the relationship?

Students use their understanding of multiplication to explain how the equation shows the relationship.

**Activity Debrief:** Ask students to share their findings.

- How did you determine what operation to use in your equations?

How students revisit the Pose the Problem question and discuss answers.

How can you describe the relationship between the number of cubes in Stick C and Stick D?

A PDF of the Teaching Resource is available in the Digital Teacher Center.

### Guided Exploration

Students use connecting cube sticks to investigate multiplicative comparison. They use the representations to formulate multiplicative relationships and make comparison statements using comparison words and equations.

### Facilitate Meaningful Discourse

- How is multiplication different from addition?
- What comparison words do we use with multiplication?
- How does the representation show multiplication?
- Have students work in pairs to formulate a comparison statement about sticks A and D. Invite pairs to share their comparison statement.
- How can we think about the relationship between Sticks A and D?
- What is the first thing we should think about?

### Math Is... Thinking

How does the equation show the relationship?

Students apply their understanding of the structure of a multiplication equation and properties of operations to explain how the equation shows the relationship.

- Think About It: How are the representations the same? How are they different?

### Develop the Math

What do you notice about the number of cubes in each of the two sticks?

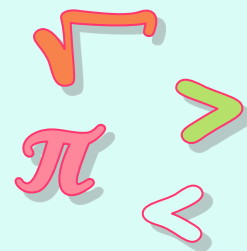
Stick A: 10 cubes  
Stick B: 20 cubes



# Reveal Math: Differentiation

The supplemental curriculum series includes the following:

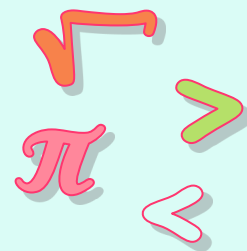
- **Tiered Practice Pages**
  - Reinforce Understanding
  - On Level
  - Extend Thinking
- **Work Stations/Centers**
  - Tiered based on reinforcing understanding, building proficiency, and extending thinking
  - Small Group Lesson Instructions & Model Activity
  - Digital Station: Gamification
  - Game Station
  - Take Another Look: Digital review & practice of concept
  - Spiral Review



<p><b>R Reinforce Understanding</b></p> <p><b>Show the Comparison</b> Provide pairs of students with pieces of paper, each piece showing a single-digit number. Also, provide index cards with more than one digit written on them. Ask students to make up problems using a starting number and a comparison that they show using the slips of paper and cards. Have them write equations to represent the problems. Make sure students identify "more than" with addition and "times" with multiplication.</p>	<p><b>B Build Proficiency</b></p> <p><b>Practice It! Game Station</b> <b>Multiplicative Comparison Drawing Task Cards</b> Students practice representing and solving problems involving multiplicative comparison.</p>	<p><b>Own It! Digital Station</b> <b>Build Fluency Games</b> Assign the digital game to develop fluency with multiplication.</p>	<p><b>E Extend Thinking</b></p> <p><b>Use It! Application Station</b> <b>The Same, But a Different Size</b> Students design square frame games and compare their areas.</p>
<p><b>Take Another Look Lessons</b> Assign the interactive lesson to reinforce targeted skills.</p> <ul style="list-style-type: none"> <li>● Interpret Multiplication Equations</li> <li>● Comparison Word Problems</li> <li>● Comparison Word Problems with Equations</li> </ul>	<p><b>Interactive Additional Practice</b> Assign the digital version of the Student Practice Book.</p>	<p><b>Spiral Review</b> Assign the digital Spiral Review practice to students or download and print PDFs of the Spiral Review from the Digital Teacher Center.</p>	<p><b>STEM Adventure</b> Assign a digital simulation to apply skills and extend thinking.</p>
<p><b>Differentiation Resource Book, p. 29</b></p> <p><b>Represent Comparison Problems</b> Name: _____</p> <p><b>Read</b> How can you use images to represent an addition or multiplication problem?</p> <p><b>Write</b> Write a number sentence for each problem. Use the words "times" or "times as many" in your sentences.</p> <p><b>Draw</b> Draw a picture to show how you solved the problem.</p>	<p><b>Student Practice Book, pp. 29–30</b></p> <p><b>Additional Practice</b> Name: _____</p> <p><b>Read</b> Read the problem. Draw a picture to show how you solved it.</p> <p><b>Write</b> Write a number sentence for each problem. Use the words "times" or "times as many" in your sentences.</p>	<p><b>Student Practice Book, pp. 30–31</b></p> <p><b>Read</b> Read the problem. Draw a picture to show how you solved it.</p> <p><b>Write</b> Write a number sentence for each problem. Use the words "times" or "times as many" in your sentences.</p>	<p><b>Differentiation Resource Book, p. 30</b></p> <p><b>Represent Comparison Problems</b> Name: _____</p> <p><b>Read</b> How can you use images to represent an addition or multiplication problem?</p> <p><b>Write</b> Write a number sentence for each problem. Use the words "times" or "times as many" in your sentences.</p>



# Reveal Math: Differentiation



The supplemental curriculum series includes the following:

- **Enrichment**
  - Application Station
  - Websketch
  - STEM Adventure
  - Career Focus
- **Fluency Practice**
  - Fluency Strategy
  - Fluency Flash
  - Fluency Check
  - Fluency Talk


### Create Color Mixtures

Paint can be a complex material. Paints are used not only to decorate, but to extend the life of the material it covers. Research to learn about the job of a paint chemist.

Imagine you are a paint chemist. Choose 3 colors of paints to use. Choose a different total of drops, 3, 4, or 6, to use for each mixture. Write a fraction for the number of drops of each color used. Compare the fractions in each row using  $>$ ,  $<$ , or  $=$ .

Once the colors are mixed, how does the greater amount of each color used affect the final mixture color? What about when the least amount of color was used? How do you think the two different color mixtures will affect the material they are painted on?

Mixture #1	Mixture #2
blue $\frac{2}{6}$	blue $\frac{1}{4}$
yellow $\frac{1}{6}$	yellow $\frac{2}{4}$
red $\frac{3}{6}$	red $\frac{1}{4}$



Paint two samples of each new color on different surfaces, such as paper, cardboard, or wood. Make two with no paint. Place all the samples in the sun for a day or longer. What effect did the sun have on those with and without paint?

- Does the paint protect the surface? Explain.
- Do certain colors in the mix increase its resistance to sun exposure? Explain.
- What important role do fractions play in the creation of colors?


Unit 4

### Multiplication as Comparison


Focus Question  
How can I compare using multiplication?

Hi, I'm Hannah.

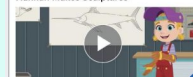
I want to be a welder! I want to make a cylinder that is welded together—just like the pipe at the playground! I need 5 times as many pieces of pipe as I already have. Let's learn to understand multiplication as a comparison.



### STEM Career: Welder



### Hannah Makes Sculptures



Unit 4

### Fluency Practice

Name: \_\_\_\_\_

**Fluency Strategy**

You can use strategies, such as doubling and breaking apart, to multiply.

You can double 2s facts to multiply by 4. You can double 4s facts to multiply by 8.

$5 \times 4 = 20$   
 $5 \times 2 = 10$ , and 4 is double 2. So,  $5 \times 4$  is double  $5 \times 2$ .  
 $5 \times 4 = 20$   
 $6 \times 3 = 18$   
 $6 \times 4 = 24$  and 8 is double 4. So,  $6 \times 8$  is double  $6 \times 4$ .  
 $6 \times 8 = 48$

You can break apart a factor to multiply by 8.

Since  $8 = 4 \times 2$ ,  $3 \times 8 = 3 \times 4 \times 2 = 24$   
 $6 \times 8 = 24 \times 2 = 48$   
 $6 \times 8 = 48$

6. What is  $9 \times 8$ ?  
 Since  $8 = 4 \times 2$ ,  $9 \times 8 = 9 \times 4 \times 2 = 36 \times 2 = 72$   
 $9 \times 8 = 72$

**Fluency Flash**

Use the model to complete the multiplication fact.

$2 \times 3 = 6$   
 $3 \times 2 = 6$   
 $3 \times 3 = 9$   
 $3 \times 4 = 12$

$4 \times 4 = 16$   
 $5 \times 4 = 20$   
 $6 \times 4 = 24$   
 $7 \times 4 = 28$   
 $8 \times 4 = 32$

Math 4 • Multiplication as Comparison • 107

### Fluency Check

What is the product or difference?

4. $3 \times 8 =$	24	11. $7 \times 4 =$	28
5. $4 \times 9 =$	36	12. $250 - 157 =$	202
6. $87 - 38 =$	49	13. $6 \times 2 =$	12
7. $7 \times 8 =$	56	14. $4 \times 6 =$	24
8. $9 \times 5 =$	45	15. $698 - 482 =$	216
9. $543 - 121 =$	422	16. $8 \times 8 =$	64
10. $4 \times 4 =$	16	17. $10 \times 6 =$	60

### Fluency Talk

How can you explain to a friend how you are breaking apart to find an 8s multiplication fact?

Explanation may vary. Sample answer: Decompose 8 to 5 and 3. Then multiply the other factor by 5 and by 3. Add the products to find the other factor times 8.

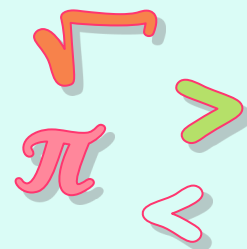
How is multiplying by 4 related to multiplying by 2?

Explanation may vary. Sample answer: If you double the 2s fact, you get the product of the 4s fact.

# Reveal Math: Assessment

The supplemental curriculum series includes the following:

- **Readiness Diagnostic Assessments**
  - Entry of Grade Level
  - Per Unit
  - Both assess students understanding of prerequisite skills required for the unit
- **End of Unit Assessments**
  - Linked to a: vocabulary review and unit review
  - Two forms of assessments
- **In Between the Unit:**
  - Daily exit tickets with student reflections
  - Performance task: Application based
  - Math Probe: Application based
- **Targeted Intervention:**
  - Links to standards, Depth of Knowledge (DOK) levels, and lesson
  - Informs teachers how to support or enrich student experiences based on progress on daily exit tickets



**Lesson 4-4 Exit Ticket**

Name \_\_\_\_\_

Which equation represents each comparison?

- 32 is 4 times as much as 8.  
 A.  $32 \div 4 = 8$       B.  $32 \times 4 = 8$   
 C.  $32 \div 4 = 8$       D.  $32 = 4 \times 8$
- 56 is 8 times as much as 7.  
 A.  $56 \times 7 = 8$       B.  $56 \div 7 = 8$   
 C.  $56 = 8 \times 7$       D.  $56 \div 8 = 7$
- Amanda has 32 pencils. She has 4 times as many pencils as markers.  
 a. Which equations could help you find how many markers Amanda has? Choose all that apply.  
 A.  $7 \times 4 = 32$       B.  $32 \div 4 = 7$   
 C.  $32 = 7 \times 4$       D.  $4 \div 7 = 32$
- How many markers does she have?  
 A. 28 markers      B. 8 markers  
 C. 6 markers      D. 4 markers
- A dozen bagels cost \$12 from the bakery. A dozen bagels cost 6 times as much as a single bagel. What is the cost of a single bagel?  
 A. \$72      B. \$6  
 C. \$2      D. \$1

**Reflect On Your Learning**


I'm confident.    I'm still learning.    I understand.    I can teach someone else.

\_\_\_\_\_

MM Assessment Resource Book

**Unit 4 How Ready Am I?**

Name \_\_\_\_\_

- What is the unknown factor?  
 $7 \times 4 = 32$   
 A. 6      B. 7      C. 4      D. 9
- Shreya buys 8 bags of apples. Each bag has 10 apples in it. How many apples does Shreya buy?  
 A. 80 apples      B. 88 apples  
 C. 90 apples      D. 100 apples
- Mali's coach makes an array of soccer balls on the practice field.  


Which equation correctly shows the total number of balls in the array?  
 A.  $3 \times 3 = 7$       B.  $3 \times 5 = 7$   
 C.  $3 = 3 \times 7$       D.  $5 \times 3 = 7$

Which number correctly completes the equation?  
 $9 \times 8 = 7$   
 A. 17      B. 63      C. 72      D. 990

Arvin puts her bubble gum into 7 groups. Each group has 6 pieces of gum. Which expression represents the number of pieces of bubble gum Arvin has?  
 A.  $7 \times 6$       B.  $9 \times 9$       C.  $7 + 7$       D.  $7 \times 9$

MM Assessment Resource Book

Item Analysis				
Item	DOK	Lesson	Guided Support Intervention Lesson	Standard
1	2	4-1	Comparison Word Problems with Equations	4.OA.A.1
2	3	4-3	Comparison Word Problems with Equations	4.OA.A.1
3a	2	4-1	Interpret Multiplication Equations	4.OA.A.1
3b				
4	2	4-2	Comparison Word Problems (Model)	4.OA.A.2
5a	3	4-3	Comparison Word Problems (Model)	4.OA.A.1
5b				
6	2	4-3	Interpret Multiplication Equations	4.OA.A.1
7	2	4-3	Interpret Multiplication Equations	4.OA.A.1
8	2	4-1	Interpret Multiplication Equations	4.OA.A.1
9a	2	4-1	Comparison Word Problems (Model)	4.OA.A.1
9b				
10	3	4-4	Comparison Word Problems with Equations	4.OA.A.2
11a	4	4-2	Solve Multiplicative Comparison Problems	4.OA.A.2
11b				
11c				

**Unit 4 Performance Task**

Name \_\_\_\_\_

**Ticket Sales**

A local theater recorded the ticket sales for its spring performances.

**Part A**

The theater sold a total of 60 tickets to its musical. That is 3 times as many tickets that were sold for the comedy. How many tickets were sold for both the musical and the comedy? Write equations with a symbol for the unknown. Draw a bar diagram to show your work.

Sample answer:  $3 \times c = m$ ;  $m = 60$ ;  $3 \times c = 60$ ;  $c = 20$ ;  $3 \times 20 = 60$ ;  $20 + 60 = 80$ . 80 comedy and musical tickets were sold. See students' bar diagrams.

**Part B**

The number of tickets sold to the comedy show is 4 times as many as the number of tickets sold to the spy show. How many spy show tickets were sold? Write an equation with a symbol for the unknown. Draw a bar diagram to show your work.

Sample answer:  $c = 4 \times s$ ;  $20 = 4 \times s$ ;  $s = 20 \div 4$ ;  $s = 5$ . 5 spy show tickets. See students' bar diagrams.

**Unit 4 Comparison Problems**

Name \_\_\_\_\_

Read each word problem. Choose of equations that represent the problem. Do not actually solve the problem.

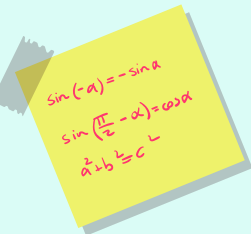
1. Ms. Olson is gathering materials to make supply boxes for her classroom. She has 9 pens. She also has 3 times as many pencils as pens. How many pencils does she have? Explain why you chose the equation or equations. Explanations may vary.

Choose all that apply.  
 A.  $9 \times 3 = 7$   
 B.  $9 \times 3 = 7$   
 C.  $9 \div 3 = 7$   
 D.  $9 \div 9 \div 9 = 7$

2. Mr. Gomez is gathering materials to make supply boxes for his classroom. He has 12 rulers. He also has 4 more erasers than rulers. How many erasers does he have? Explain why you chose the equation or equations. Explanations may vary.

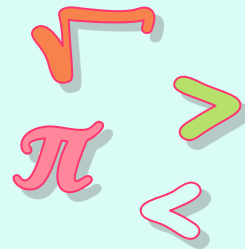
Choose all that apply.  
 A.  $12 \times 4 = 7$   
 B.  $12 \div 4 = 7$   
 C.  $12 \div 4 = 7$   
 D.  $12 + 12 + 12 = 7$





$\sin(-a) = -\sin a$   
 $\sin\left(\frac{\pi}{2} - a\right) = \cos a$   
 $a^2 + b^2 = c^2$

# Reveal Math: Supports



The supplemental curriculum series includes the following:

- **English Learner Scaffolds**
  - Aligned to WIDA levels
  - Provides small group instruction based on the following levels:
    - Emerging/Entering
    - Developing/Expanding
    - Bridging/Reaching
  - Materials (Print & Digital) are in Spanish
  - Digital platform can be changed to any language with Google Translate extension

## English Learner Scaffolds

**Entering/Entering** Support students' understanding of the expression so. Using counters, put two piles of 10 together. Gesture to one pile and say *ten*. Gesture to the next pile and say *ten*. Say, *I have ten plus ten counters. So, I have twenty counters.* Put another two piles together (10 and 20) and say, *I have 10 plus 20 counters. So how many do I have?* (30). Continue practicing with other sets.

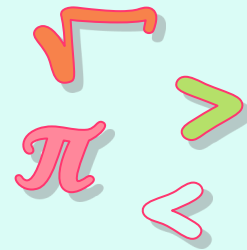
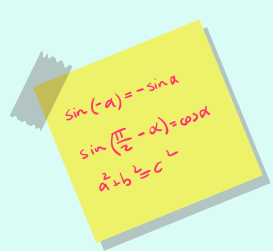
**Developing/Expanding** Support students' understanding of the expression so. Using counters, put two piles of 10 together. Gesture to one pile and say *ten*. Gesture to the next pile and say *ten*. Say, *I have ten plus ten counters. So, I have twenty counters.* Put another two piles together (10 and 20) and say, *I have 10 plus 20 counters. So how many do I have?* (You have 30 counters.)

**Bridging/Reaching** Engage students in a discussion about other ways to say *I have ten plus ten counters, so I have twenty*; for example, *If I have ten counters and ten counters, then I have twenty.*









# Number Worlds Authorships

The following expert authors participated in the formulation of the program:

- **Sharon Griffin, Ph.D:** Cognitive science to improve mathematics learning and achievement for at-risk children
- **Douglas Clements, Ph.D:** Scholar in the field Early Childhood Learning & “scaling up” interventions
- **Julie Sarama, Ph.D:** Conducts research on young children’s development of mathematical concepts and competencies with focus on scaling-up educational reform



$$\sin(-\alpha) = -\sin \alpha$$

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos \alpha$$

$$a^2 + b^2 = c^2$$



# Number Worlds Leveling Guides

The overarching goal of the assessments used to determine specific levels of each individual child is to determine where the child's mathematical misconceptions start. Therefore, those concepts can be addressed and the teacher can formulate targeted, tiered lessons for each individual child that works back up to the grade level standard.

\*\*\*A combined score between the two assessments listed below is utilized to determine the specific level of each individual child

**Number Knowledge Test:** Assesses students based on their number sense and fact fluency knowledge

**Placement Test:** Assesses students based on their conceptual understanding per grade level

**The Number Knowledge Test Record**

Total Score		New Test Score	
Name	D.O.B.	Parent Age Score	
Date		Grade Level	
School		NW Level	
Preliminary Count 1-10			
Level 0 (2 or more correct, go to next level)		Strategy	Score
1. Count (1)	2b. More: 3 vs. 7		
2a. More: 5 vs. 2	3b. Less: 8 vs. 3		
3a. Less: 2 vs. 8	4b. Count 8 (6)		
4. Count 8 (6)	5b. Count 8 (6)		
5. Count 8 (6)			
Level 1 (5 or more correct, go to next level)		Strategy	Score
1. 4 + 3 =	2b. More: 3 vs. 7	CU CO R	
2. 7 + 1 =	3b. Less: 8 vs. 3		
3. 7 + 2 =	4b. Bigger: 7 or 9		
4a. Bigger: 5 or 6	5b. Smaller: 5 or 7		
5a. Smaller: 5 or 6	6b. Closer to 7: 4 or 9		
6a. Closer to 5: 6 or 2	7b. Closer to 7: 4 or 9		
7. 2 + 4 =	8b. Closer to 7: 4 or 9	CU CO R	
8. 8 - 6 =	9b. Last: 8 5 2 6	CU CO R	
9a. First: 8 5 2 6			
Level 2 (5 or more correct, go to next level)		Strategy	Score
1. 40 - 5 =	2b. Bigger: 32 or 28		
2. 60 - 4 =	3b. Smaller: 17 or 12		
3a. Bigger: 69 or 31	4b. Smaller: 3 or 24		
4a. Smaller: 17 or 12	5b. Closer to 25: 20 or 30		
5a. Closer to 25: 20 or 30	6b. Closer to 28: 31 or 24		
6. How many numbers between 2 and 8	7b. Closer to 28: 31 or 24		
7. How many numbers between 7 and 9	8b. Closer to 28: 31 or 24		
8. 12 + 54 =	9b. Closer to 28: 31 or 24	CO	
9. 42 - 21 =			
Level 3		Strategy	Score
1. 99 + 10 =	2b. Bigger: 6 - 2 or 8 - 5		
2. 89 + 9 =	3b. Smaller: 99 - 82 or 25 - 11		
3a. Bigger: 6 - 2 or 8 - 5	4b. Smaller: 99 - 82 or 25 - 11		
4a. Smaller: 99 - 82 or 25 - 11	5b. Closer to 48: -36 or 84 - 73		
5a. Closer to 48: -36 or 84 - 73	6b. Closer to 48: -36 or 84 - 73		
6. 36 - 18 =	7b. Closer to 48: -36 or 84 - 73		
7. 301 - 7 =			
* Correct answers item			
32 Number Knowledge Test Record			

Name \_\_\_\_\_ Date \_\_\_\_\_

Circle the letter of the correct answer.

8. What number is missing in this function table?

IN	OUT
3	8
5	10
7	
15	20
23	28

A 33 B 14  
C 29 D 12

9. Which multiplication fact is represented by this array?




A 5 x 4 = 20 B 6 x 4 = 24  
C 8 x 3 = 24 D 6 x 5 = 30

10. Which equation is correct?

A 5 + 9 = 14 - 5  
B 5 + 5 = 9 + 9  
C 5 + 9 = 5 + 5  
D 9 + 5 = 14 - 9







- **Learning Targets/Objectives**
- **Standards Alignment:** NJSLS
- **Vocabulary:** Based on researched tiers of vocabulary instruction- Uses Frayer Model Word Wall Cards
  - Mathematical terms
  - Academic terms
  - English Language Learners supports
- **Lesson Structure:**
  - Warm-Up Activity: Linked to Number Sense
  - Develop: Mini-lesson
  - Alternative Groups: Helps teachers adapt instruction to fit the needs of various learners
  - Daily Progress Monitoring: allows teachers to adapt instruction to provide immediate feedback and targeted instruction

► **Then...** distribute paper clips to students so that they can model the amount on their cards along the edges.

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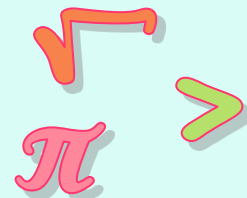
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$$\sin(-\alpha) = -\sin \alpha$$

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos \alpha$$

$$a^2 + b^2 \leq c^2$$

# Number Worlds: Academic Focus



The supplemental curriculum series includes the following:

- **Lesson Structure:**
  - Differentiation:
    - Independent Practice & Game Based Learning
    - Supported Practice
- **Assessments**
  - Number Knowledge & Placement Test
  - Project Based Learning: Weekly standards-driven project based learning activities to support long-term retention of concepts
  - Daily Exit Tickets
  - Weekly Assessments
  - Unit Assessments
- **Digital Platform: Gamification**
  - Building Blocks

## Building Blocks Activities

Support and motivate students with an adaptive, personalized learning system that is proven to work. With **Building Blocks**, students are actively engaged in their learning as they progress through adaptive math activities.

**Building Blocks**, the result of NSF-funded research, develops students' mathematical thinking through interactive, web-based practice activities. Students progress through research-proven learning trajectories, making connections and effectively building mathematical understanding.



## Strategic Digital Modeling

Students and teachers have access to a variety of web-based digital math tools. These powerful tools support reasoning skills and problem solving by allowing students to virtually explore and model mathematical concepts.



Name \_\_\_\_\_ Date \_\_\_\_\_ Unit 9

**Test**

1. 9 18

2. + =

3. 12 16

4. + =

Build the Math Teacher 28

Project evaluation criteria rubrics allow teachers and students to discuss aspects of the project and expectations for completing the project.

**Project Evaluation Criteria**

Knowing project evaluation criteria with students prior to beginning the project.

**Research Expectations**

- Project result is explained and can be extended.
- Project result is explained in context and can be applied to other situations.
- Project result is explained using advanced mathematical vocabulary.
- Project result is described, and mathematics are used correctly and can be extended.
- Project result is explained and extended, and shows advanced knowledge of mathematical concepts and skills.

**Maths Expectations**

- Project result is explained.
- Project result is explained in context.
- Project result is explained using mathematical vocabulary.
- Project result is described, and mathematics are used correctly.
- Project result is explained and shows satisfactory knowledge of mathematical concepts and skills.

**Does Not Meet Expectations**

- Project result is not explained.
- Project result is explained, but not in context.
- Project result is described, but mathematical vocabulary is inappropriate.
- Project result is not explained and there are some inconsistencies.
- Project result is not explained and there are some inconsistencies but demonstrates a lack of understanding of mathematical concepts and skills.

**Introduce**

- Read the story and listen to the audio. You can read and listen to it again if you like.
- Answer the questions on your project to explain to students and teachers how you think you can be a scientist.
- How can you think you can be a scientist? (Write your answer in the space provided.)
- How can you think you can be a scientist? (Write your answer in the space provided.)

**Explore**

- Read the story and listen to the audio. You can read and listen to it again if you like.
- Answer the questions on your project to explain to students and teachers how you think you can be a scientist.
- How can you think you can be a scientist? (Write your answer in the space provided.)
- How can you think you can be a scientist? (Write your answer in the space provided.)

**Wrap Up**

- Read the story and listen to the audio. You can read and listen to it again if you like.
- Answer the questions on your project to explain to students and teachers how you think you can be a scientist.
- How can you think you can be a scientist? (Write your answer in the space provided.)
- How can you think you can be a scientist? (Write your answer in the space provided.)

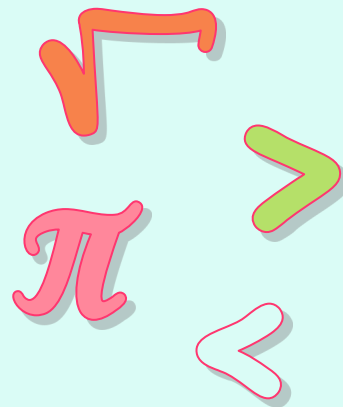
Build the Math Teacher 29

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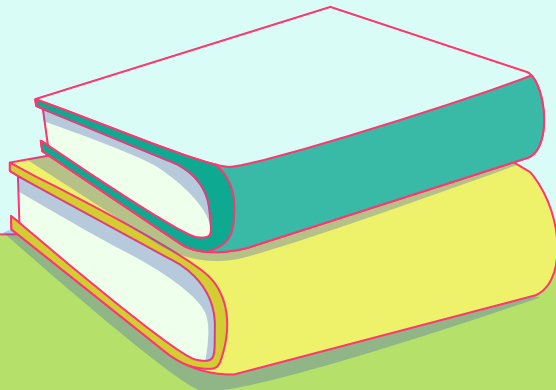
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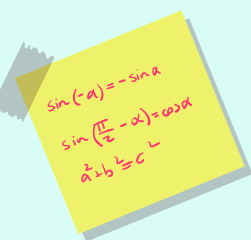
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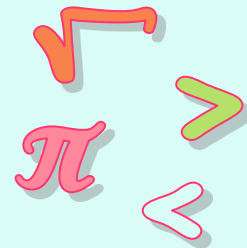
# 3 Year Plan





$\sin(-a) = -\sin a$   
 $\sin\left(\frac{\pi}{2} - a\right) = \cos a$   
 $a^2 + b^2 = c^2$

# Three Year Plan



The topics below are ones that represent the future goals for the mathematics department:

- **2022-2023 School Year:** Gain familiarity with the new supplemental curriculum series in the first year of implementation & gain all levels of support
- **2023-2024 School Year:**
  - Grades K-2: Study and utilize the Addition/Subtraction Graham Fletcher Kits to support fluency strategy based small group instruction
    - Differentiating between skill based (concept/standard aligned) and strategy based (fluency) small group instruction
  - Grades 3-5: Study the conceptual foundations of fractions and support instruction of fractions within the upper elementary grade levels
    - I.e. equivalent fractions, comparing fractions, adding and subtracting fractions (all kinds), multiplying and dividing fractions (all kinds), and fractional real world application problems
- **2024-2025 School Year:** Study, analyze, and implement math conferences
  - Will work in alignment to Jennifer Serravallo conferences in Reading



# Thank You!

Let's work together to prepare  
our children of today for  
tomorrow!



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